LEVEL II

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Correction Factors for a Reciprocity Calibration

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The theory for correcting the voltages measured during a reciprocity calibration is developed. The correction factor pertaining to the reciprocal transducer has not been utilized previously, but it must be considered when the capacitance of the interconnections in relation to the capacitance of the transducer exceeds the desired accuracy. Experimental verification of the theory is presented. A method for determining the equivalent capacitance of the interconnections and measurement system is also discussed.

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CORRECTION FACTORS FOR A RECIPROCITY CALIBRATION

INTRODUCTION

Reciprocity theory in electroacoustic systems was initially discussed by Schottky [1]. Ballantine [2] and MacLean [3] have shown how the theory, using three transducers, can be applied for the absolute calibration of electroacoustic devices by purely electrical measurements. Numerous authors have elaborated on this calibration technique and its applications.

Conventional application of the three-transducer method requires one transducer (S) to be used only as a source, one (D) to be used only as a receiver, and a reciprocal transducer (T) to be used as both.[a] The measurement technique employed is shown schematically in Fig. 1.

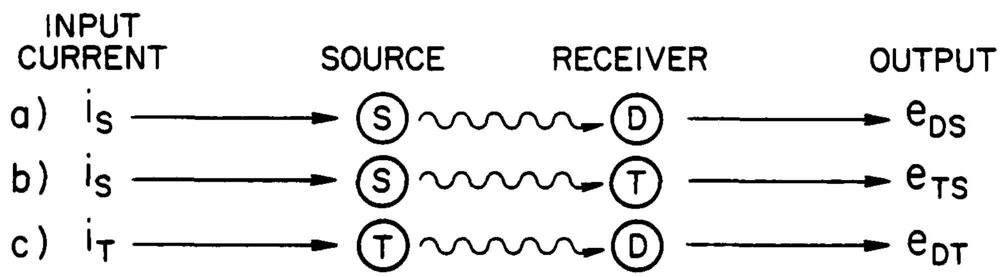

Fig. 1 - Diagram of the measurements required for a reciprocity calibration

The free-field voltage sensitivity, M_D, of the receiver is determined using the following equation. (For the derivation, the reader should see Reference [4].)

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[a]The reader is referred to the List of Symbols for an explanation of terms used in this article.

Note: Manuscript submitted January 17, 1979.

$$M_D = [J \cdot e_{DT}\, e_{DS}/(e_{TS}\, i_T)]^{\frac{1}{2}} \tag{1}$$

The measured quantities required by Eq. (1) are ideal measurements, unaltered by the measurement system. However, the introduction of a measurement system and the interconnections will always perturb the desired parameters. The purpose of this article is to review the necessary correction factors for a precise calibration using a reciprocity technique.

LIST OF SYMBOLS

The three elements used in reciprocity calibrations are denoted by symbol subscripts: D for the detector, T for the reciprocal transducer, and S for the source. A list of the symbols used follows.

e_{KL} - open-circuit voltage produced at the terminals of element K when element L is transmitting

e'_{KL} - actual voltage indicated when the measurement system is attached

e''_{KL} - actual voltage measured with the known capacitance, C_o, added to the system

e_T - voltage produced across the known resistance, R_o, when element T is transmitting

e'_T - actual voltage measured across R_o by the system

E_o - constant voltage driving element T when it is transmitting

i_T - current flowing through element T when it is transmitting

f - frequency in Hz

j - $\sqrt{-1}$

J - reciprocity parameter - see reference [4]

M_D - free-field open-circuit receiving sensitivity of element D in volts/Pa

M'_D - receiving sensitivity of element D using only the receiver correction factor

M''_D - receiving sensitivity of element D using both the receiver and transducer correction factors

ω - angular frequency ($2\pi f$)

Z_{MK} - complex impedance of measurement system input, including interconnections $[R_M/(1+j\omega R_M C_{MK})]$, when connected to element K

Z_o - known complex impedance added in parallel to the measurement system

R_o - known resistance used to measure transducer current

C_K - capacitance of element K

C_o - known capacitance added in parallel to the measurement system

C_{MK} - capacitance of measurement system input, including interconnections, when connected to element K

THEORY

If each receiving element, K, is subjected to a constand sound-pressure magnitude, $\bar{p}$, due to the transmitting element, L, the receiver may be represented as shown in Fig. 2. By Thevin's theorem,

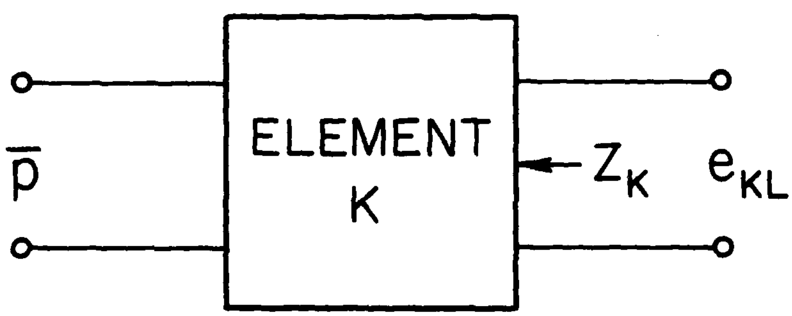

Fig. 2 - Schematic representation of a transducer subjected to a sound pressure amplitude (magnitude)

the element representation in Fig. 2 can be replaced by that in Fig. 3.

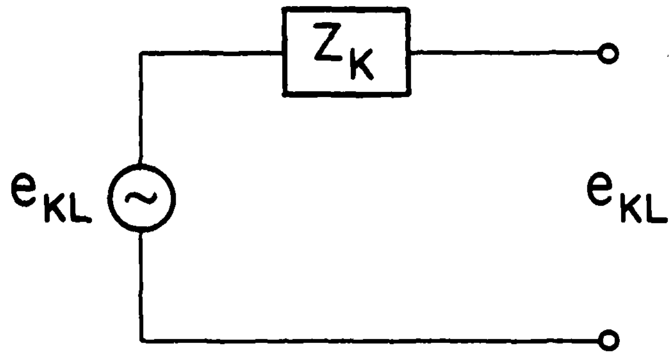

Fig. 3 - Thevin representation of Fig. 2

When a measurement system is connected to the output terminals, the voltage measured would not be e_{KL} because the system will alter the circuit as shown in Fig. 4. From this circuit, the following equation can be derived.

$$e_{KL} = e'_{KL}\ Z_K\ (1/Z_K + 1/Z_{MK}) \tag{2}$$

This equation can be applied to all the required receiver voltage measurements.

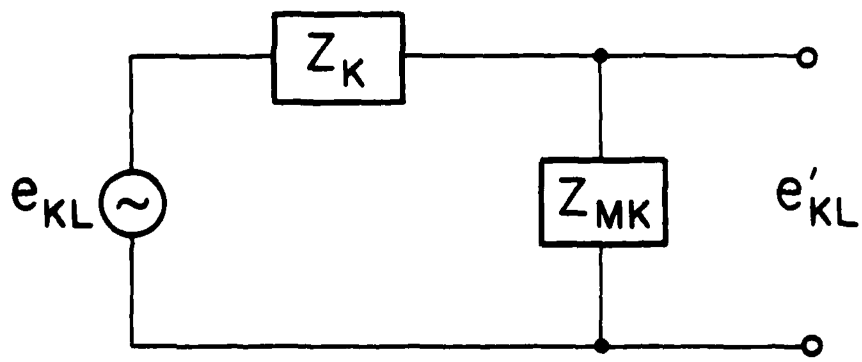

Fig. 4 - Schematic of the transducer in Fig. 3 when the interconnections and measurement system are connected

The current i_T in Eq. (1) can be determined by measuring the voltage produced across a known resistance, R_o, when the transducer T is transmitting. The Thevinin equivalent of this circuit is shown in Fig. 5.

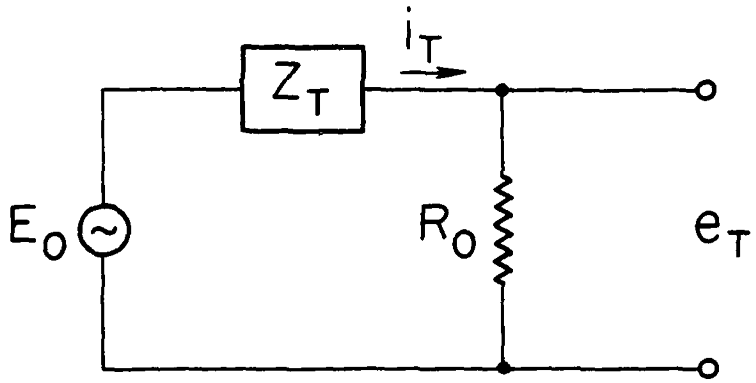

Fig. 5 - Thevinin representation of a transducer when driven by a constant-voltage generator and acting as a source

When the measurement system is connected, the circuit can be represented as shown in Fig. 6.

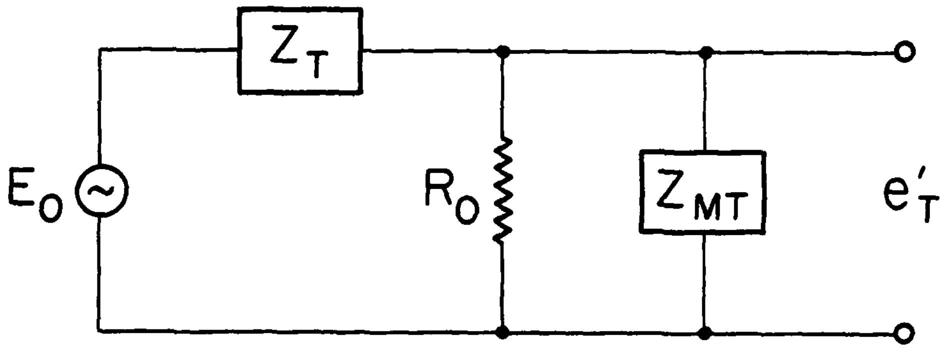

Fig. 6 - Schematic of the transducer in Fig. 5 when the interconnections and measurement system are connected

The following equation can be derived from the last two circuits.

$$i_T = [e'_T/R_o] \{1 + R_o Z_T/[Z_{MT}(R_o + Z_T)]\} \qquad (3)$$

If the proper forms of Eqs. (2) and (3) are substituted into Eq. (1),

$$M_D = \left(\frac{e'_{DS}e'_{DT}}{e'_{TS}e'_T}\, JR_o\right)^{\frac{1}{2}} \frac{[1 + Z_D/Z_{MD}][Z_{MT}(R_o + Z_T)]^{\frac{1}{2}}}{\{[1+Z_T/Z_{MT}][Z_{MT}(R_o+Z_T) + R_o Z_T]\}^{\frac{1}{2}}} . \qquad (4)$$

Assuming the impedances of all elements can be represented as pure capacitances and the impedance of the measurement system to be $\dfrac{R_M}{1 + j\omega C_{MK}R_M}$, Eq. (4) becomes

$$M_D = \left(\frac{e'_{DS}e'_{DT}}{e'_{TS}e'_T} JR_o\right)^{\frac{1}{2}} \frac{C_T^{\frac{1}{2}}[C_D+C_{MD}+1/j\omega R_M)][R_M/1+j\omega R_M C_{MT})]^{\frac{1}{2}}}{C_D[C_T+C_{MT}+1/(j\ R_M)]^{\frac{1}{2}}[R_M/(1+j\ R_M C_{MT})+R_o/(1+j\ R_o C_T)]^{\frac{1}{2}}} . \qquad (5)$$

If $R_M >> R_o$, Eq. (5) reduces to the following:

$$M_D = \left(\frac{e'_{DS}e'_{DT}}{e'_{TS}e'_T}\, JR_o\right)^{\frac{1}{2}} \left[\frac{C_D + C_{MD} + 1/(j\omega R_M)}{C_D}\right] \left[\frac{C_T}{C_T + C_{MT} + 1/(j\omega R_M)}\right]^{\frac{1}{2}} . \qquad (6)$$

In many cases involving reciprocity-couplers and high input-impedance amplifiers, $\frac{1}{\omega R_M}$ is negligible when compared to either C_D or C_T. Then Eq. (6) reduces to the following form.

$$M_D = \left(\frac{e'_{DS}e'_{DT}}{e'_{TS}e'_T}\, JR_o\right)^{\frac{1}{2}} \left(\frac{C_D + C_{MD}}{C_D}\right) \left(\frac{C_T}{C_T + C_{MT}}\right)^{\frac{1}{2}} \qquad (7)$$

The first factor is the same as the standard reciprocity calibration except that measured voltages are used instead of open-circuit voltages. The second factor is the correction commonly used to adjust the sensitivity of a receiver when cable is added. This is usually the only correction applied to receiver calibrations. The last factor is the correction required by the addition of interconnections and the measurement system to the reciprocal transducer. To the knowledge of the author, the reciprocal-transducer correction factor has not been used previously.

EXPERIMENTAL RESULTS

Verification of these correction factors was conducted experimentally using a three-transducer reciprocity coupler developed at the Naval Research Laboratory's Underwater Sound Reference Detachment (NRL/USRD) [4]. Data were obtained utilizing the method illustrated in Fig. 1. Numerous sets of data were obtained for various temperatures and pressures. The results of a typical set of data are shown in Table I.

TABLE I. Receiving sensitivity of a transducer for two system capacitances as a function of the calibration equation

	Receiver Sensitivity (dB re 1 Volt/μPa)	
	Case 1	Case 2
M'_D	-211.67	-210.86
M''_D	-211.16	-210.34
M_D	-213.04	-213.07

Case 2 measurements were conducted in the same manner as Case 1, except a known capacitance was added to the reciprocal-transducer cable. M'_D was calculated using only the first factor of Eq. (7), M''_D using the first two factors, and M_D using all of the factors. The values for the capacitances are shown in Table II.

TABLE II. Values of capacitances used in calculating Table I

	Capacitance (pF)	
	Case 1	Case 2
C_S	5306	5306
C_T	398.8	398.8
C_D	2156	2156
C_{MS}	155.9	155.9
C_{MT}	216.5	347.8
C_{MD}	130.5	130.5

DISCUSSION

Further investigation of the correction factor theory yields a technique for measuring the equivalent capacitance of the interconnections and the measurement system. The capacitance can be determined without disturbing the calibration arrangement by adding a known capacitance to the system.

If a known impedance Z_o is added in parallel to the circuit in Fig. 4, it can be easily shown that:

$$e_{KL} = e''_{KL}\ Z_K(1/Z_K + 1/Z_{MK} + 1/Z_o), \tag{8}$$

where e''_{KL} is the new measured voltage.

Solving Eqs. (2) and (8),

$$Z_{MK} = Z_o Z_K(e''_{KL}-e'_{KL})/[Z_o(e'_{KL}-e''_{KL})-Z_K e''_{KL}]. \tag{9}$$

Assuming that the impedances can be represented as pure capacitances,

$$C_{MK} = [e''_{KL}C_o/(e'_{KL}-e''_{KL})]-C_K. \tag{10}$$

If C_K has been measured previously and C_o is known, the magnitude of the capacitance for the interconnections and measurement system can be obtained by measuring the voltage e_{KL} with and without the capacitor C_o.

Because the term $1/(j\omega R_M)$ is included in Z_{MK} and may not be negligible, the calculated value of C_{MK} may be frequency dependent and should be determined at each frequency for which reciprocity calibrations are conducted. The same fact can be applied to the other impedances if their resistive components are not negligible compared to the capacitances. Also, C_o should be chosen such that inaccuracies are not introduced by the subtraction in the denominator; a value for C_o which is approximately $9(C_K + C_{MK})$ should alleviate this problem.

CONCLUSIONS

The reciprocity-calibration technique was devised as an absolute calibration of a transducer. It has been shown in Table II that large errors (>2 dB) can occur in the receiving sensitivity of the transducer if incomplete correction factors are applied to the measured data. The theory, which has been presented with experimental verification, provides insight into the problem and one method for correcting the errors. The techniques required for implementation of the correction factors and the capacitance measurements can be readily incorporated into existing systems.

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